MA189. *Proposed by Alaric Pow Ian-Jun.*

Find the range of values of the constant k such that the equation

$$(x+1)(x+3)(x+5)(x+7) = k$$

has 4 distinct solutions for x.

We received 10 submissions, of which 8 were correct. We present (with minor amendments) the solution by the Missouri State University Problem Solving Group.

Letting $x = u - 4$ and expanding, the equation becomes

$$u^4 - 10u^2 + 9 = k.$$

By the quadratic formula, the solution to this equation is

$$u = \pm\sqrt{5 \pm \sqrt{16+k}}.$$

If the intent was for x to be real, then we must have $16 + k \geq 0$, i.e., $k \geq -16$. Since $k = -16$ only gives two values for u, it must be rejected. We must also have $5 - \sqrt{16+k} \geq 0$ or equivalently $k \leq 9$. Since $k = 9$ only gives three values for u, it must also be rejected. Thus the range for k is $-16 < k < 9$.

If the intent was for x to be complex, then $k = -16$ and $k = 9$ are the only values that give fewer than four roots.